EELE 250: Circuits, Devices, and Motors

Lecture 13

Assignment Reminder

- Read 5.1 – 5.4 AND 5.5-5.6
- Practice problems:
  - P5.23, P5.24, P5.35, P5.37
  - P5.42, P5.44, P5.47, P5.49, P5.57

- D2L Quiz #6 by 11AM on Monday 10 Oct.

- REMINDER: Lab #3 report is due and Lab #4 will be performed this week—be sure to do the pre-lab assignment calculations!

- Exam #2: in class on Monday 17 Oct.

A few impedance questions…

- An inductor of value 0.5 henry is used in a circuit driven by a source v(t) = $V_m \cdot \cos(200t)$.

The impedance of the inductor is:

(a) j(50/π) ohms

(b) j(100/π) ohms

(c) j100 ohms

(d) j200 ohms

(e) none of the above

A few impedance questions…

- A 10 uF capacitor is used in an AC steady-state circuit. At what radian frequency is the magnitude of its impedance equal to 250 Ω ?

(a) 10 rad/sec
(b) 400 rad/sec
(c) 250 rad/sec
(d) 250π rad/sec
(e) 800 π rad/sec

A few impedance questions…

- What is the equivalent impedance of the network "seen" by the source?

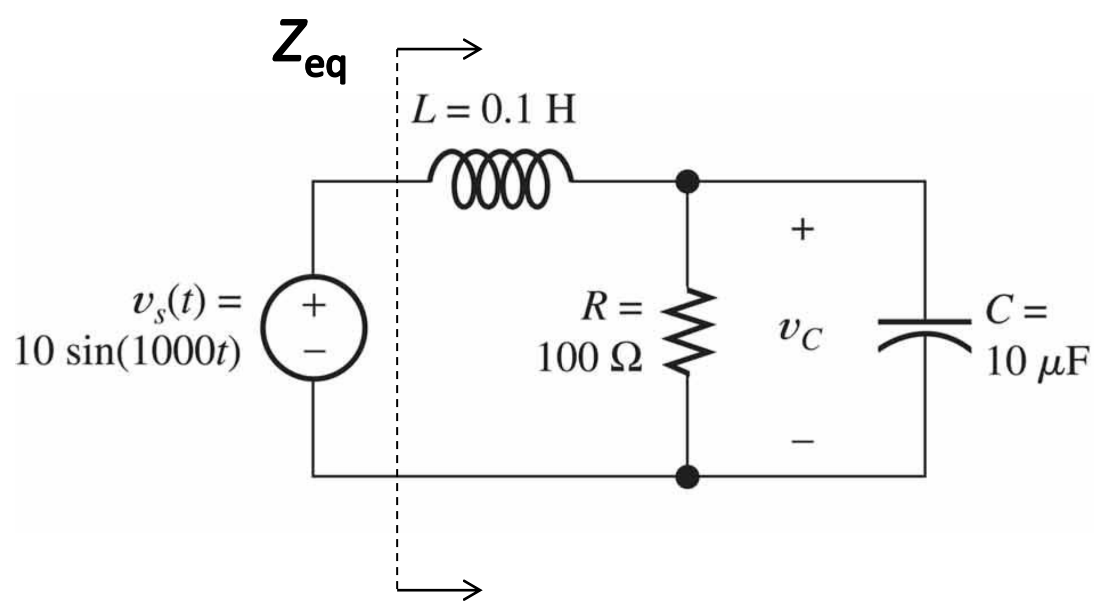
(a) 100 Ω

(b) j100 Ω

(c) 100 + j100 Ω

(d) 150 Ω

(e) 50 + j50 Ω



Power in AC Circuits

- Power is the rate at which energy is used.
- watts = joules/second
- volts x amps = (joules/coulomb)x(coulomb/sec)=watts

Resistive Load

- Let $v(t) = V_m \cdot \cos(\omega t)$
- For a resistor, V=IR, so $i(t) = I_m \cdot \cos(\omega t)$

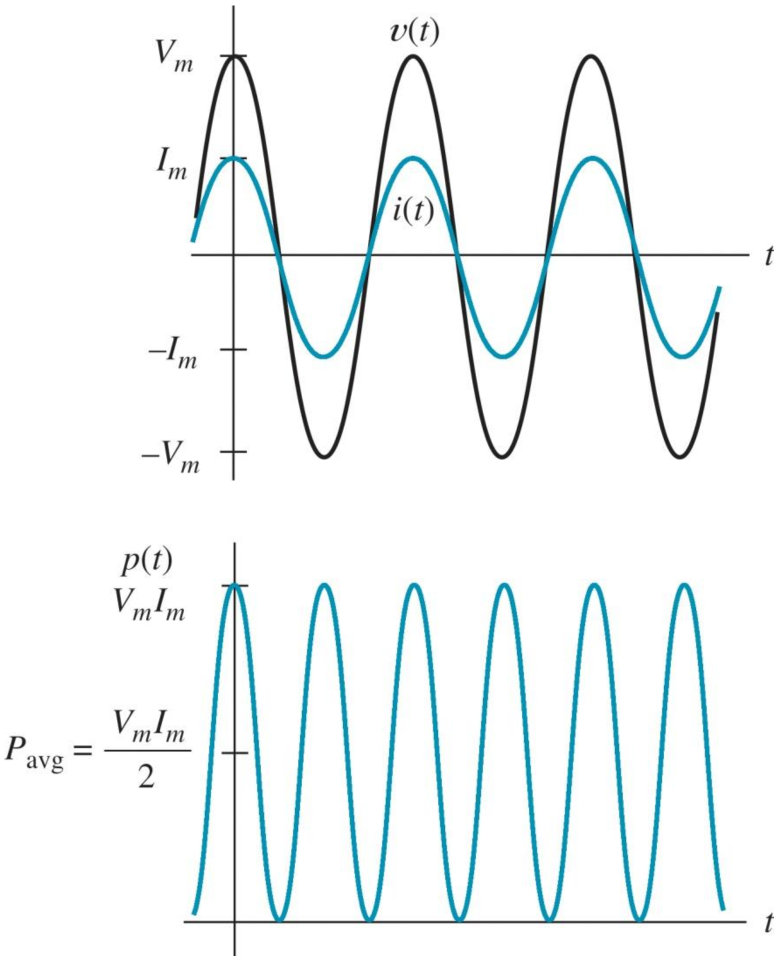
  $(I_m = V_m/R)$
- $p(t) = v(t) \cdot i(t) = V_m \cdot I_m \cdot \cos^2(\omega t)$

- Note that *v* and *I* are *in phase* and $p(t) \geq 0$

Resistive Load (cont.)

Current and voltage are in phase

Power is always non-negative

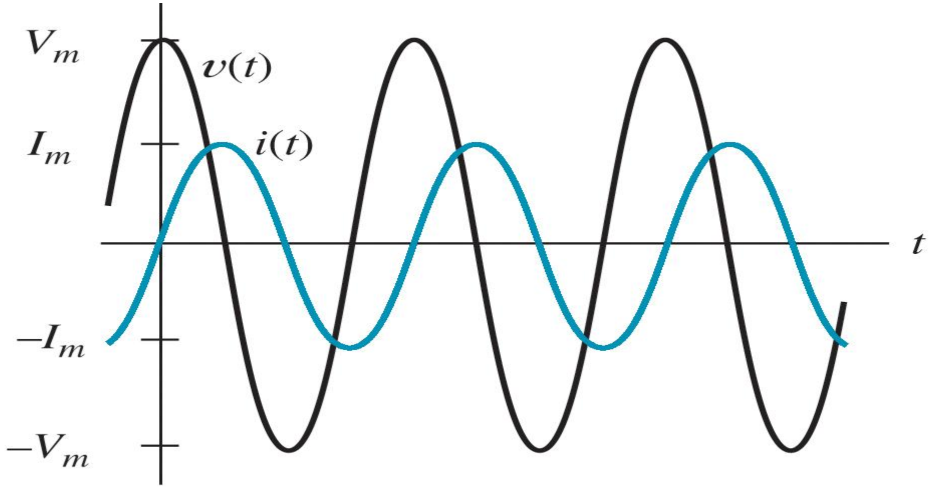


Inductive Load

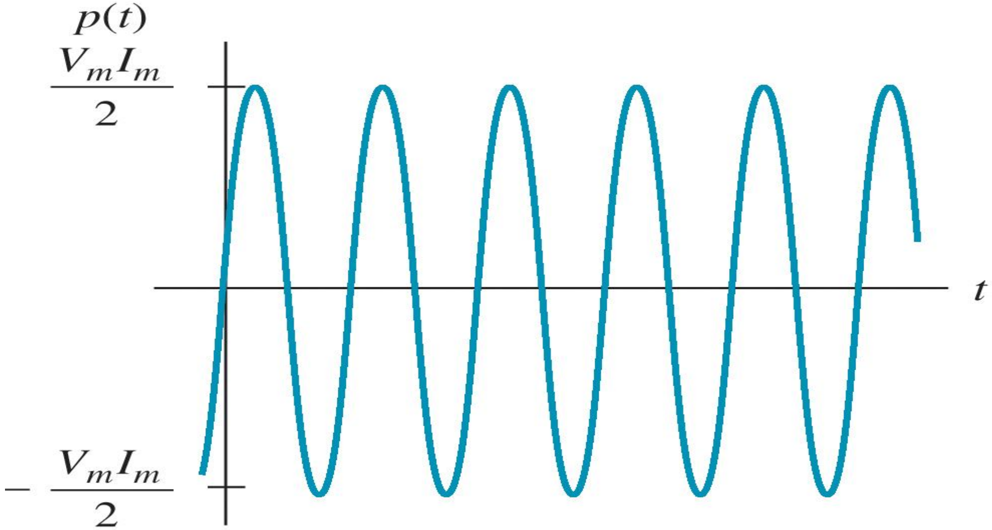
- Let $v(t) = V_m \cdot \cos(\omega t)$ $\quad\quad$ $\mathbf{V} = V_m \angle 0°$
- For an inductor, $\mathbf{Z} = j\,\omega\,L = \omega L \angle 90°$
- $\mathbf{I} = \mathbf{V}/\mathbf{Z} = (V_m/\omega L) \angle{-90°}$
- $i(t) = I_m \cdot \cos(\omega t - 90°) = I_m \cdot \sin(\omega t)$

  $(I_m = V_m / \omega L)$
- $p(t) = v(t) \cdot i(t) = V_m \cdot I_m \cdot \cos(\omega t) \cdot \sin(\omega t)$
- Note that *v* and *i* are *out of phase* and p(t) is both positive and negative

Inductive Load (cont.)



Current *lags* voltage



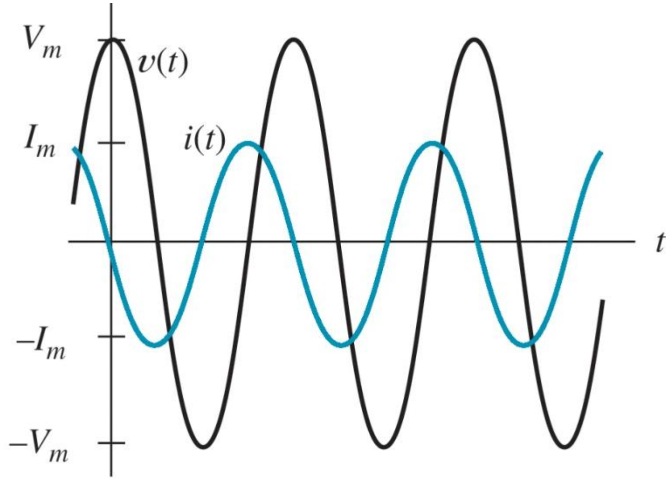
Power is positive *and* negative

Capacitive Load

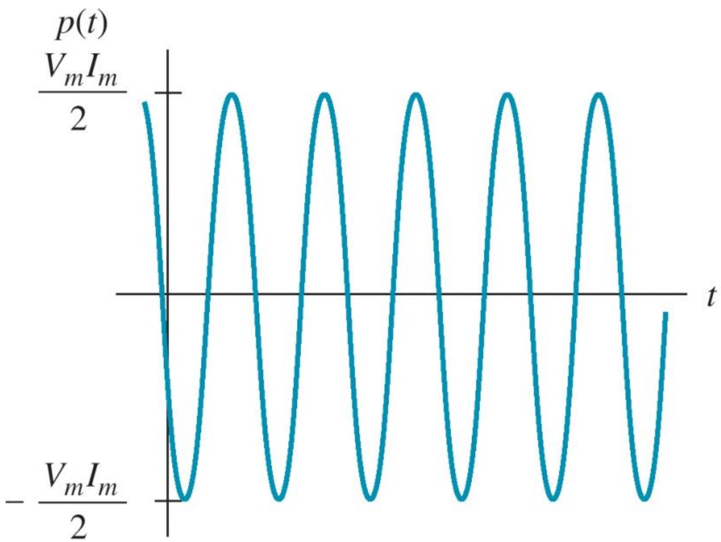
- Let $v(t) = V_m \cdot \cos(\omega t)$ $\mathbf{V} = V_m \angle 0°$
- For a capacitor, $\mathbf{Z} = 1/(j\omega C) = 1/(\omega C) \angle -90°$
- $\mathbf{I} = \mathbf{V}/\mathbf{Z} = (V_m \omega C) \angle 90°$
- $i(t) = I_m \cdot \cos(\omega t + 90°) = -I_m \cdot \sin(\omega t)$

  $(I_m = V_m \omega C)$
- $p(t) = v(t) \cdot i(t) = -V_m \cdot I_m \cdot \cos(\omega t) \cdot \sin(\omega t)$
- Note that *v* and *i* are *out of phase* and p(t) is both positive and negative

Capacitive Load (cont.)



Current *leads* voltage



Power is positive *and* negative

Power for general RLC load

- In general, let

  $v(t) = V_m \cdot \cos(\omega t)$ and $i(t) = I_m \cdot \cos(\omega t - \theta)$
- And thus $p(t) = V_m I_m \cdot \cos(\omega t) \cos(\omega t - \theta)$

  which can be re-written as

$$p(t) = (1/2)V_m I_m \cdot \cos(\theta)[1+ \cos(2\omega t)]$$
$$+ (1/2) V_m I_m \cdot \sin(\theta)\sin(2\omega t)$$

- The *average* power: $P = (1/2)V_m I_m \cdot \cos(\theta)$

Power for general RLC load (cont.)

- P = (1/2)$V_m\, I_m \cdot \cos(\theta)$ can be written as

  P = $V_{rms}\, I_{rms} \cdot \cos(\theta)$,

since for sinusoids rms = amplitude / sqrt(2)

- Recall that θ is the angle by which the current lags the voltage: $\theta_v - \theta_i$

  v -> cos(ω*t*)   *i* ->cos(ω*t* - θ)

- cos(θ) is called the *power factor*.

Reactive Power and Power Factor

- The power factor gives an indication of average power delivery to the load.
- For a resistive load, $\theta = 0$, so $\cos(\theta) = 1$
- For a purely capacitive or inductive load, $\theta = \pm 90°$, so $\cos(\theta) = 0$
- The power that flows in and out of a load is called the *reactive power*, *Q*.

$$Q = V_{rms} I_{rms} \cdot \sin(\theta),$$